4 Preliminary

The Lotka-Volterra model describes a deterministic competition system[3] between prey & predator populations denoted by $N_1(t)$ & $N_2(t)$ respectively where $a, b, c, g > 0$.

$$\mathbf{\frac{d(N_1(t))}{dt} = -cN_1(t) + gN_1(t)N_2(t)} \tag{5}$$

$$\mathbf{\frac{d(N_2(t))}{dt} = aN_2(t) - bN_1(t)N_2(t)} \tag{6}$$

We look at a spatial adaptation of such a model where the range of interaction between these two populations is restricted[7]. The formulation in Eqn 7 & Eqn 8 is consistent with that of[7] where $\bar{x}, D_{N_i}$ represents the spatial location (x, y) & the diffusion coefficient of preys,predators respectively. Further, prey's grow with a rate r & are consumed by predators at a rate of α. Predators on the other hand die at a rate of m, & reproduce at the rate β. The spatial range between these two populations R_i affects their interaction terms.

$$\mathbf{\frac{\partial N_1(\bar{x},t)}{\partial t} = D_{N_1}\frac{\partial^2 N_1(\bar{x},t)}{\partial t^2} + rN_1(\bar{x},t) - \alpha N_1(\bar{x},t)\int_{|\bar{x}'-\bar{x}|<R_1} N_2(\bar{x}',t)d\bar{x}'} \tag{7}$$

$$\mathbf{\frac{\partial N_2(\bar{x},t)}{\partial t} = D_{N_2}\frac{\partial^2 N_2(\bar{x},t)}{\partial t^2} - mN_2(\bar{x},t) + \beta N_2(\bar{x},t)\int_{|\bar{x}'-\bar{x}|<R_2} N_1(\bar{x}',t)d\bar{x}'} \tag{8}$$

We simplify the limiting interaction integral based on the series solution to the probability of a point (x, y) lying within a circle of radius R, where the coordinates are distributed according to a bivariate normal by a multiple of the Incomplete Gamma Function where $z_i = \frac{R_i}{\sigma_x}$. Further, we assume $\sigma_x = \sigma_y = \sigma$, allowing us to approximate the Incomplete Gamma function[18].

Subsequently we adapt this model to derive it's stochastic counterpart by including independent Brownian motion terms $\omega_1(t)$ & $\omega_2(t)$[3;11] to obtain Eqn 9 & Eqn 10.

$$\mathbf{\frac{\partial N_1(\bar{x},t)}{\partial t} = D_{N_1}\frac{\partial^2 N_1(\bar{x},t)}{\partial t^2} + rN_1(\bar{x},t) - \alpha N_1(\bar{x},t) z_2 e^{\frac{-1}{2(b^2+z_2^2)}} I_0(bz_2) + G_1 N_1(\bar{x},t)\omega_1(t)} \tag{9}$$

$$\mathbf{\frac{\partial N_2(\bar{x},t)}{\partial t} = D_{N_2}\frac{\partial^2 N_2(\bar{x},t)}{\partial t^2} - mN_2(\bar{x},t) + \beta N_2(\bar{x},t) z_1 e^{\frac{-1}{2(b^2+z_1^2)}} I_0(bz_1) + G_2 N_2(\bar{x},t)\omega_2(t)} \tag{10}$$

where :
$z_i \geq 0$
I_0 = Modified Bessel function of the First Kind & Zero order
$b = \sqrt{\frac{\mu_x^2}{\sigma_x^2} + \frac{\mu_y^2}{\sigma_y^2}}$

We hence obtain a spatial extension to the typical stochastic formulation shown in[3], where a linear estimation of the same is derived & thus conclude our competition system can be modelled using a linear system of equations.